Name ____________________________________ ______ raw ______ scaled ______ percent

Math 11 Trimester 3 Exam 2 (402 Points)

Cumulative Final Exam

- **This is a take home exam. Here are the rules:**

  The exam is due by the end of the scheduled final exam period at 10:15 on Friday, June 6.

  You may
  [1] use your book, your notes, and a calculator while doing the exam,
  [2] use any other book while doing the exam,
  [3] use the internet to learn more about these topics while doing the exam (not recommended).

  You may not
  [1] communicate with anyone about these questions until the exams have all been collected. This includes communicating in person, in writing, over the phone, on-line.

- **Any questions about these rules, just ask me at any time. If you believe there is an error in a question, ask me about it.**
- **Please work out your solutions as rough drafts on paper other than this exam paper. When you turn in this exam (on this paper) it should be your final draft of your best work. If you need another blank copy of this exam, just ask me or download one from my web site.**
- **To receive full credit, provide a complete solution. Answers must be exact and fully simplified.**
- **Answers must be completely simplified. No denominators may include radicals. All fractions reduced. Arithmetic must be completely performed; e.g. write 9 instead of $\sqrt{81}$ and $2\sqrt{3}$ instead of $\sqrt{12}$.**

■ **A. Evaluate the following. (7 points each)**

[1] $\sqrt[4]{16}\ \sqrt[3]{16}\ \sqrt[6]{16}$

[2] $125^{\frac{2}{3}}$

[3] $\left(3^{\sqrt{7}}\right)^{-\pi} \cdot 3^{\sqrt{7\pi^2}}$

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■ **B. Answer the following. (10 points each)**

[1] Let $f : f(x) = \frac{3x+5}{(x-2)}$. State the domain of f (the largest set of numbers for which f is defined) and the range of f.

[2] Let $f : f(x) = \frac{3x+5}{\sqrt{x-2}}$. State the domain of f (the largest set of numbers for which f is defined) and the range of f.

[3] Let $f : f(x) = x^2$ and $g : g(x) = x + 3$. Evaluate $f(g(x))$ at $x = 2$.

[4] Find functions f and g such that $f \circ g = h$, for $h : h(x) = (x^2 + 4x - 3)^{\frac{1}{2}}$.

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[5] Suppose that $f : f(x) = x^2$ for $x \in (0, \infty)$. Circle the letter of each statement that is true.

a. f is a 1-1 function.

b. f has an inverse function.

c. The range of f is the set of all real numbers.

d. f has an inverse function and the domain of its inverse function is $\mathbb{R}$.

e. f is an increasing function of x.

■ C. Solve the following for x. (10 points each)

[1] $8^x = 4$

[2] $9^{x+1} = 27^x$

[3] $3^x \cdot 9^{2x} - 27 = 0$

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[4] $3^{x^2+x-6} = 1$

[5] **Let** $f : f(x) = a^x,\ 0 < a < 1$. **What is the range of** f**?**

■ **D. Answer each of the following. (12 points each)**

[1] **Write** $\log_5 125 = 3$ **in exponential form.**

[2] **Write** $5^2 = 25$ **in logarithmic form.**

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[3] Solve for y. $\log_2 8 = y$.

[4] Solve for y. $\log_y 81 = 4$.

[5] Solve for y. $\log_3 y = 2$.

[6] Solve for y. $\log_{\sqrt{3}} y = 6$.

[7] Solve for y. $\log_y 36 = -2$.

[8] If $\log_2 5 = 2.322$, what does $\log_2 5^2$ equal?

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[9] Rewrite as a single logarithm. $3\log_{10}(x-2) - 2\log_{10}$

[10] Solve for x. $\log_2(x^2 + 3x + 4) = 1$.

All angles you write for answers must be written with respect to the angle zero and measured in the positive direction (counter clockwise). For example, write $\theta = \frac{3\pi}{2}$ rather than $\theta = -\frac{\pi}{2}$.

■ **E1. Fill in the table below. ($\frac{3}{4}$ point each cell)**

[1]

$\theta \longrightarrow$	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$	2π
sin (θ)								
cos (θ)								

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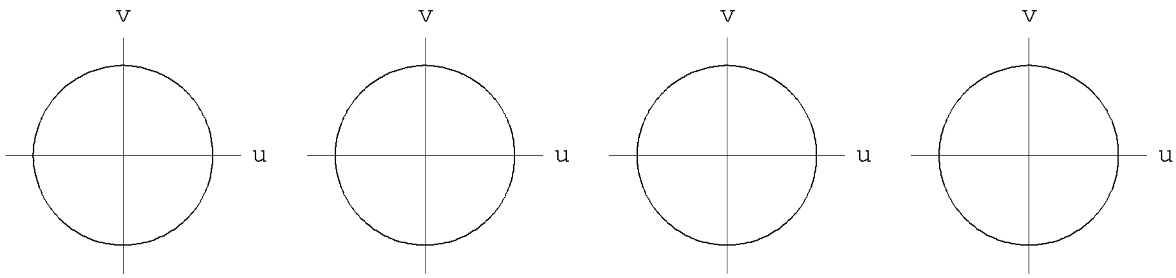
■ **E2. Find the following. (12 points each)**



[2] cos $\frac{-7\pi}{6}$

[3] sin $\frac{11\pi}{6}$

[4] tan $\frac{-4\pi}{3}$

[5] Find all x, $0 \leq x \leq 2\pi$, for which cos $x = \frac{-1}{2}$

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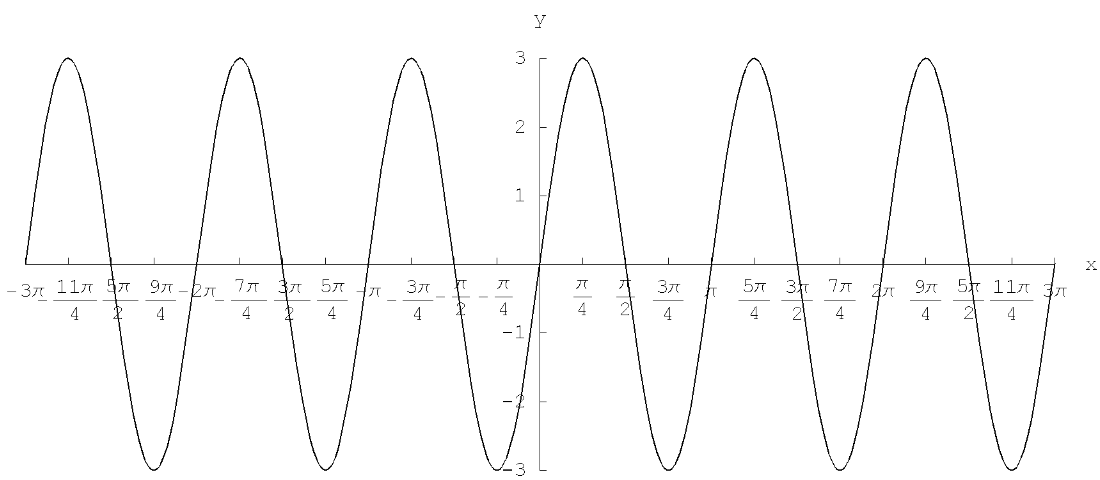
[6] $\cos \frac{5\pi}{12}$ (you probably need to use an identity for this one)

■ **E3. Answer the following. (12 points each)**

[1] The graph crosses the x-axis at …, $-\pi$, $\frac{-\pi}{2}$, 0, $\frac{\pi}{2}$, π, ….
It has a maximum of 3 and a minimum of -3.



The function whose graph is shown is:

Answer

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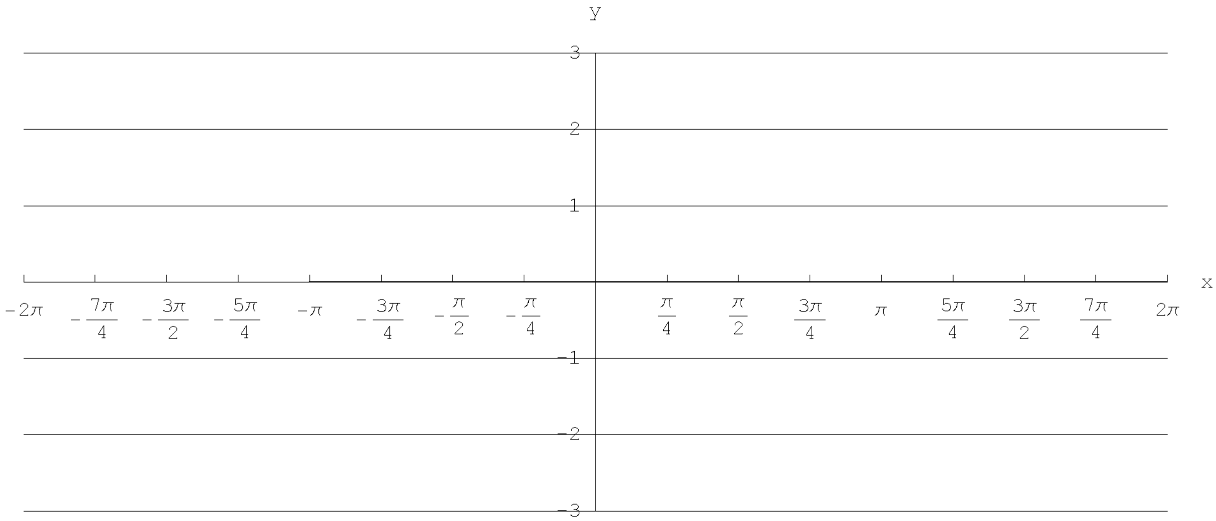
[2] $y = 2\sin\left(x - \frac{3\pi}{4}\right)$



State three values of x where the graph of $y = 2\sin\left(x - \frac{3\pi}{4}\right)$ intersects the x-axis.

1.

2.

3.

[3] State all asymptotes of the tangent function.

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■ **F1. Answer the following. (7 points each)**

[1] The sequence $\frac{3}{2}$, 3, $\frac{9}{2}$, 6, $\frac{15}{2}$, 9, … is

a. **arithmetic.**
b. **geometric.**
c. **<u>neither</u> arithmetic and <u>nor</u> geometric.**

■ **F2. Write the first 4 terms of each sequence whose general term is below. (7 points each)**

[2] $a_n = (-1)^n\ 2^n$

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[3] Find the least number of terms of the geometric sequence *4, 8, 16, 32, …* which must be taken for their sum to exceed *1020*. (Handy formula: $S_n = a\,\frac{1-r^n}{1-r}$)

■ G. Proofs. (10 points each)

[1] $1 + \tan^2 \phi = \frac{1}{\cos^2 \phi}$

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[2] Prove any <u>one</u> of the following - your choice.

a. Prove that the function defined by $g: g(x) = x^n$, $n \in \{1, 3, 5, 7, \ldots\}$ is a 1-1 function.

b. Prove: $\tan(x - y) = \frac{\tan x - \tan y}{1 + \tan x \tan y}$.

c. Prove that for the geometric series $S_n = a + a r + a r^2 + \ldots + a r^{n-2} + a r^{n-1} = \frac{a(1-r^n)}{1-r}$

d. Prove that the sum, S, of the infinite geometric series $a + a r + a r^2 + \ldots$, where $|r| < 1$, is equal to $\frac{a}{1-r}$. That is, $S = \frac{a}{1-r}$, $|r| < 1$.